University of Texas Bulletin

No. 2163: November 10, 1921

The Texas Mathematics Teachers' Bulletin

Volume VII, No. 1

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SECOND-CLASS MATTER AT THE POSTOFFICE AT AUSTIN, TEXAS,
UNDER THE ACT OF AUGUST 24, 1912

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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The Texas Mathematics Teachers' Bulletin

Volume VII, No. 1

Edited by

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Instructor in Pure Mathematics,

and

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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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CONTENTS

MATHEMATICS FACULTY OF THE UNIVERSITY OF TEXAS

JOINT MEETING OF THE MATHEMATICS SECTION OF THE TEXAS STATE TEACHERS' ASSOCIATION AND THE TEXAS SECTION OF THE MATHEMATICAL ASSOCIATION OF AMERICA

On Friday and Saturday, November 25 and 26, there will be held in Dallas a joint meeting of the Mathematics Section of the Texas State Teachers' Association and the Texas Section of the Mathematical Association of America. Professor G. C. Evans of Rice Institute is chairman of the former body. The latter group was organized last Thanksgiving at the Fort Worth meeting of the State Teachers' Association, with Professor H. J. Ettlinger of the University of Texas as chairman and Professor J. L. Riley of the John Tarleton Agricultural College as secretary. Every mathematics teacher in the state is strongly urged to attend these meetings and to take part in the program. These meetings will be held in the Bryan High School building, in conjunction with the other section meetings of the State Teachers' Association. A joint dinner is also being planned for Friday night.

FAULT IN FACT AND FANCY IN MATHEMATICS—WITH SOME SUGGESTED CORRECTIONS

T. M. Broadfoot

Is the fault with the text or the teacher that our pupils are so often promoted without a proper understanding of the fundamentals of mathematics? This is a question that always presents itself to my mind when reading of reformed mathematics with its hoped for and promised curative effects upon the mathematical ills of our educational system.

It seems to be the spirit of the day among many mathematics teachers, great and small, that our course is top-heavy with quantity and that this is the most serious hindrance we have to securing the best quality of work. There is also the feeling among many, that reformed mathematics will completely suffice in the removal of these ills.

Now, there are two things that are always of greater importance to the teacher than the textbook, namely the study of the child and the study of the teacher herself. The teacher must be thoroughly familiar with her subject, of course, and she must be versatile in her methods of presentation. These she must continually study from the retrospective as well as from the prospective point of view. Then she must strive to know the child's mental activities in order to know his difficulties and how to guide him over them by variations in her methods. Always upon the discovery of something going wrong in my own classes, it has been my custom to begin the investigation at home, and it has often been found unnecessary to go beyond my own gates in making corrections. This method has also been found, to a certain degree, applicable to teachers under my supervision. In either case, defects of whatever nature or degree have been sought there and new methods applied before seeking to place the burden of responsibility upon the text or the child. This course has often brought immediate relief in mathematics as well as in other subjects, and will, I believe, if consistently and watchfully followed, bring relief for most of the ills of the classroom.

It is not the purpose of this paper to offer defense of the texts, or the treatment of subjects, now in use in mathematics, nor to argue that quantity has nothing to do with the quality of work done, but that, while the texts may be somewhat faulty and be in need of reform, our methods are often also too limp and need wise revision.

Now, before our methods can properly be determined, we must have a clear understanding of the problems that confront the child. Truly, each child has its own individual problems. These the teacher must learn through personal contact, and herein lies one of the teacher's most sacred duties. But there are some problems that are fundamental to the thinking process, and thus belong to all children. With these the teacher should become familiar. One of the most far-reaching of these lies in the fact of whether the pupil has or has not learned that the universe is composed of relations as well as facts. These are two distinct stages of mental development, and the transition from the one to the other is a most difficult and a most treacherous one. It is with the birth of this transition that mathematics has the peculiar distinction of being connected. It is in the transition that every teacher's chief concern should lie, and it is in this that our methods have the greatest need for modification.

The modifications I would suggest are that we should apply a little more freely the principles that many of us have either largely forgotten or have not quite realized the full importance of in the learning process, those principles in which so many authors have placed so much value, namely drill and correlation. It is my belief, based upon a number of years of experimentation in the practical psychology of the classroom, that, for the large majority of students, these are two of the most important principles ever announced in the psychology of learning.

The question now comes, how are we to apply them with the proper thoroughness, and cover the amount of ground we have to cover? How can we afford to spare the time to be devoted to drill more than the text allows in the review exercises? And it is often true that this is insufficient. I can well remember these questions in my own experience and how to depart from the letter of the text seemed the unpardonable sin of pedagogy. But I have since learned that text books are written, not to be used merely as the rod that bears the vane, but as the vane itself which may be twirled and twisted to suit every wind that blows. It is placed in the hands of the teacher to be used, not as an iron-clad limitation, but as a suggestive outline which is to direct her in her efforts to mould the imperfect minds of her pupils into perfect form. The teacher who is wise to the task before her will make the text pliable enough in her hands to fit the conditions of all minds in her charge and especially to apply to that large majority of pupils who need constant drill to make memory stick and vigilant aid in correlation to make the thinking process complete, that is, to enable them to see the relation that one fact bears to another.

Review exercises alone will not accomplish this desirable result; nor will a separate review of lessons previously studied; neither will the extension of the time given to the simpler problems, as seems to be the chief remedy offered by the reformists. My favorite practice has been to use these in modified form and in combination, in such a way as to secure the proper amount of drill and correlation with the least expenditure of time and the minimum exhaustive effect upon the child's enthusiasm, that is, through review of correlated subjects previously studied in connection with any new division of the work taken up. This method brings to the teacher's aid the impetus furnished by the pupil's new realization of necessity, brought about by his having studied the more difficult problems based upon these. Now comes the psychological moment, in his mental development, for the teacher to make a wise use of correlation. Take, for example, an assignment in algebra containing problems like the following: to find the quotient by inspection of

$\dfrac{9a^4-25y^4}{3a^2+5y^2}$. Explain, or have explained, that the numerator, being the difference of two squares, is derived by one of the special rules of multiplication, and ask the class to give that rule and tell of what two factors the numerator is composed, letting them see that the denominator is one of those factors. Without this method, many pupils will spend considerable time dividing the numerator by the denominator, especially if the problem be one of several terms, rather than spend the time to learn the new method by which he is to be taught to let the mind reach back from the new to the old fact, discovering for himself the relation that exists between the old and the new. If he is permitted to omit this, he is permitted to overlook the main purpose for which he studies mathematics and is irreparably crippling his mental development. Again in factoring, when the assignment includes the trinomial a perfect square, as: $9x^2+24xy+16y^2$, show that it has the form of the square of the sum of two numbers, what the special characteristics of that form are, and that this sum may be found by adding the square roots of the first and last terms, etc. A few problems under the special rules should be assigned in connection with each of these lessons, making sure that those assigned bear the proper relation to the new assignment, until all pupils have become familiar with the true relations.

A second method, which I have used very effectively in eliminating quantity, has been to assign only the odd or even numbers of problems, sometimes only every alternate odd or even numbers, especially in very long lists and where the problems are of fairly uniform type. This carries the advantage of an assignment of problems sufficiently well graded as to difficulty, and, in many instances, also enables the teacher to eliminate the work, in excess of that necessary to a sufficient acquaintance with the subject-matter, which so often stagnates the interest of the pupils before they have quite gained the desired skill in manipulation. If at the end of the list their information is found insufficient, the other numbers may be given on the pretext of review. This always brings additional zeal since it puts the class in touch with more ease because of their easier grade and of the pupils' added strength of insight and manipulation.

In case their information and ability to solve are still too limited, I have followed still a third course: that is, under pretext of resting on that list of particularly difficult problems, we move on to the next list, usually a new subject and a new process, with a distinct understanding that, as soon as we have rested, we are to return to the mastery of the old list. I have always found the class to give a sigh of relief at this information and to return later with renewed vigor and determination after having discovered the real need of the method left unmastered. Time for this review is often more than gained from the many lists that contain more problems than are found necessary.

These same methods may be applied in modified form, with equal advantages, in geometry and trigonometry; but to conserve space, their strict applications to these subjects will be left to the ingenuity of the teacher, if there be any who have not already applied them. I shall mention, however, one other expedient that has been found quite effective in eliminating the confusion of facts and truths of trigonometry, brought about by their too rapid accumulation in an effort to cover so much ground in so short a time. This plan is to cover the required ground in about fourteen weeks, seeking a fair understanding of the fundamentals and their application, excepting as to the formulas. These we seek only to memorize and to explain the different steps at this time. During the remainder of the semester, we combine the review with a study to master all fundamentals and the development of all formulas. This process might be reversed and, from one point of view, the reverse would seem the more logical; but I have found this one gives the better satisfaction, because more psychological.

It is not that I claim any originality in the above combinations, nor that I wish to boast of any special accomplishments in their use that the following statement is made.

It is done to impress more deeply, if possible, the place and importance of the devices and plans herein set forth. Numbers of teachers, whom I have had the pleasure to teach in the normals, have come to me at the end of the term with the sentiment of the following direct quotation: "You have shown me a new light in the teaching of mathematics. I see it more clearly myself, and believe I shall realize better results from my own teaching."

Too few teachers feel that pupils need much assistance in correlation or even in drill; but in all high school subjects in general, and in mathematics in particular, the teacher's chief duty lies in this work. The longer I remain in the schoolroom the more importance I attach to it as a living principle in my professional philosophy. It is a great necessity, because the average pupil, at this stage of his mental development, has neither formed the habit nor realized the importance of carrying forward into any new subject, or phase of the same subject, the fundamental facts and principles of the old and knitting them with the new into a harmonious whole. He continues to adhere to the practice of childhood, of regarding all facts as individual, having no relation whatever to other facts. In other words, he has not yet recognized the one great truth, that, so far as man's thinking process is concerned, all facts are in some way related to other facts and all truths grow out of other related truths, or he at least failed to learn first how these relations are discovered; and it is at this point in his mental development that he most needs the assistance of a wise and well-informed instructor. This is one of the most important and one of the first lessons every teacher should learn. To the extent that she has left it unlearned and to the extent that she has left it unrevealed and unimpressed, her efforts at teaching are bound to result in failure. There is no better nor quicker way to do this important work for the pupil than by drill and correlation through review as herein set forth—drill, that he may have within the storehouse of memory an ample supply of facts that may be related; and correlation, that he may know how to discover those relations. For this purpose let us practice what has been found by our superior predecessors to be good. In mathematics let us live close to the authors of methods.

[Mr. Broadfoot is Principal of the Weatherford High School.]

ELECTIVE COURSES IN MATHEMATICS FOR SECONDARY SCHOOLS

A PRELIMINARY REPORT BY THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS

INTRODUCTION

The Committee has elsewhere expressed its judgment that in the seventh, eighth and ninth grades mathematics should be a required subject. In the tenth, eleventh and twelfth grades, however, the extent to which election of subjects is permitted will depend on so many factors of a general character that it seems unnecessary and inexpedient for the present Committee to urge a positive requirement beyond the minimum one for the seventh, eighth and ninth grades. The subject must, like others, stand or fall on its intrinsic merit or on the estimate of such merit by the authorities responsible at a given time and place. The Committee believes, nevertheless, that every standard high school should not merely offer courses in mathematics for the tenth, eleventh and twelfth grades, but should encourage a large proportion of the pupils in its general courses to take some or all of these courses. Apart from the intrinsic interest and great educational value of the study of mathematics, it will in general be necessary for those preparing to enter college or to engage in the numerous occupations involving the use of mathematics to do work beyond the minimum requirement.

The present report is intended to suggest the most valuable mathematical training for students in general courses in secondary schools that will be supplementary to the first courses in mathematics as outlined in a previous report of the National Committee.* Under present conditions most of this work will normally fall in the last two years of the high school, i.e., in general, in the eleventh and twelfth school years.

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*Cf. Secondary School Circular No. 5 (February, 1920), U. S. Bureau of Education.

The selection of material is based on the following general principles:†

1. The primary purpose of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and of space which are necessary to a better appreciation of the progress of civilization and a better understanding of life and the universe about us and to develop those habits of thinking which will make these powers effective in the life of the individual.

2. The courses in each year should be so planned as to give the pupil the most valuable mathematical information and training which he is capable of receiving in that year with provision for his vocational and later educational needs.

The second principle leads us to the conclusion that the material for the elective courses offered should include, as far as possible, those mathematical ideas and processes that have the most important applications in the modern world. As a result we will naturally include certain material that at present is not ordinarily given in secondary courses; as, for instance, the material concerning the calculus. On the other hand we shall exclude certain other material that is now included in college entrance requirements. The results of an investigation made by the National Committee in connection with a study of these requirements indicates that modifications to meet these changes will be desirable from the standpoint of both college and secondary school.‡

One can not too strongly emphasize the fact that the broadening of content of high school courses in mathematics

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†The first principle is adopted unchanged from the report just referred to; the second principle is a modified form of the second principle of the same report. This modification is a logical consequence of the somewhat different aim of elective courses in mathematics as compared with courses for all students; for after students have completed the minimum essentials of mathematics it will be found desirable to differentiate their training according to special life aims, interests or aptitudes in so far as these are discoverable.

‡Cf. "College Entrance Requirements in Mathematics," a preliminary report, to appear in the May, 1921, number of the *Mathematics Teacher*.

suggested in the report on the first courses and in the present report will very materially increase the usefulness of these courses to those who pursue them. It is of prime importance that educational administrators and others charged with the advising of students should take careful account of this fact in estimating the relative importance of mathematical courses and alternative elections. The number of important applications of mathematics in the activities of the world is today very large and is increasing at a rapid rate. This aspect of the progress of civilization has been noted by all observers who have combined a considerable knowledge of mathematics with an alert interest in the newer developments in other fields. It was revealed in very illuminating fashion during the recent war by the insistent demand for persons with varying degrees of mathematical training for many war activities of the first moment. Other forms of special training were also in demand, but in no single instance was the demand so widespread. If the same effort were made in time of peace to secure the highest level of efficiency available for the specific tasks of modern life, the demand for those trained in mathematics would be no less insistent. For it is in no wise true that the applications of mathematics in modern warfare are relatively more important or more numerous than its applications in those fields of human endeavor which are of a constructive nature.

There is another important point to be kept in mind in considering the relative value to the average student of mathematical courses and various alternative courses. If the student who omits the mathematical courses has need of them later, it is almost invariably more difficult and it is frequently impossible for him to obtain the training in which he is deficient. In the case of a considerable number of alternative subjects a proper amount of reading in spare hours at a more mature age will ordinarily furnish him the approximate equivalent to what he would have obtained in the way of information in a high school course in the same subject. It is not, however, possible to make up deficiencies in mathematical training in so simple a fashion. It requires systematic work under a competent teacher to master properly the technique of the subject and any break in the continuity of the work is a handicap for which increased maturity rarely compensates. Moreover, when the individual discovers his need for further mathematical training, it is usually difficult for him to find the time from his other activities for systematic work in elementary mathematics.

RECOMMENDATIONS FOR ELECTIVE COURSES

The following topics are recommended for inclusion in the mathematical offerings to pupils who have satisfactorily completed the work outlined in the National Committee's report on The Reorganization of the First Courses in Secondary School Mathematics, comprising the elementary notions of Algebra, Intuitive Geometry, Numerical Trigonometry and Demonstrative Geometry.

1. ALGEBRA. (a) *Simple functions of one variable.* Numerous illustrations and problems involving linear, quadratic and other simple functions including formulas from science and common life. More difficult problems in variation than those included in the earlier course.

(b) *Equations in one unknown.* Various methods for solving a quadratic equation (such as factoring, completing the square, use of formula) should be given. In connection with the treatment of the quadratic a very brief discussion of complex numbers should be included. Simple cases of the graphic solution of equations of degree higher than the second should be discussed and applied.

(c) *Equations in two or three unknowns.* The algebraic solution of linear equation in two or three unknowns and the graphic solution of linear equations in two unknowns should be given. The graphic and algebraic solution of a linear and a quadratic equation and of two quadratics that contain no first degree term and no xy term should be included.

(d) *Exponents, radicals and logarithms.* The definitions of negative, zero and fractional exponents should be given and it should be made clear that these definitions must be adopted if we wish such exponents to conform to the laws for positive integral exponents. Reduction of radical expressions to those involving fractional exponents should be given as well as the inverse transformation. The rules for performing the fundamental operations on expressions involving radicals, and such transformations as

$$\sqrt[n]{a/b} = \frac{1}{b}\sqrt[n]{ab^{n-1}}, \quad \sqrt[n]{a^n b} = a\sqrt[n]{b}, \quad \frac{a}{\sqrt{b}+\sqrt{c}} = \frac{a(\sqrt{b}-\sqrt{c})}{b-c}$$

should be included. In close connection with the work on exponents and radicals there should be given as much of the theory of logarithms as is involved in their application to computation and sufficient practice in their use in computation to impart a fair degree of facility.

(e) *Arithmetic and Geometric Progressions.* The formulas for the nth term and the sum of n terms should be derived and applied to significant problems.

(f) *Binomial Theorem.* A proof for positive integral exponents should be given; it may also be stated that the formula applies to the case of negative and fractional exponents under suitable restrictions, and the problems may include the use of the formula in these cases as well as in the case of positive integral exponents.

2. SOLID GEOMETRY. The aim of the work in solid geometry should be to exercise further the spatial imagination of the student and to give him a knowledge of the fundamental spacial relationships and power to work with them. For many of the practical applications of mathematics it is of fundamental importance to have accurate space perceptions. Hence it would seem wise to have at least some of the work in solid geometry come as early as possible in the mathematical course, preferably not later than the beginning of the third year of high school. For schools that can complete more than the preliminary courses outlined in the previous report during the first two years it would seem that the more elementary notions of solid geometry might well be listed among the optional topics of the first two years to be studied in connection with related ideas of plane geometry.

The work in solid geometry should include numerous exercises in computation based on the formulas established. This will serve to correlate the work with arithmetic and algebra and to furnish practice in computation. It is of first importance that such practice be continued throughout the entire mathematical course. For a minimum course it will be possible to omit a considerable number of propositions ordinarily given and many of the exercises of a more theoretical nature. For example, proofs of the more difficult propositions dealing with volumes and areas, that can be established more readily by the methods of the calculus, may well be postponed until they can be discussed as applications of the latter subject. The minimum course should certainly include propositions dealing with perpendiculars to planes, dihedral angles, and the simpler theorems on areas and volumes. It should be possible to complete such a minimum course in a third of a year.*

3. TRIGONOMETRY. The work in elementary trigonometry begun in the earlier years should be completed by including the logarithmic solution of right and oblique triangles, radian measure, graphs of trigonometric functions, the derivation of the fundamental relations between the functions and their use in providing identities and in solving easy trigonometric equations. There should be, wherever practicable, some use of the transit in connection with the simpler operations of surveying and of the sextant for some of the simpler astronomical observations, such as those involved in finding local time. When no transit or sextant is available, simple apparatus for measuring angles roughly may and should be improvised. Drawing to scale should form an essential part of the numerical work in trigonometry. The use of the slide rule in computations requiring

*Some European schools have found it desirable to replace some of the work now usually given in solid geometry by certain important topics of descriptive geometry. Since no textbook is at present available for this purpose the Committee refrains from any recommendation in this direction. The possibility of a scientific and logical treatment of descriptive geometry would seem to be worthy of the attention of teachers, however.

only three place accuracy and in checking other computations is also recommended.

4. ELEMENTARY STATISTICS. Continuation of the earlier work to include the meaning and use of fundamental concepts and simple frequency distributions with graphic representations of various kinds and measures of central tendency.

5. ELEMENTARY CALCULUS. The work should include:

(a) The general notion of a derivative as a limit indispensable for the accurate expression of such fundamental quantities as velocity of a moving body or slope of a curve.

(b) Applications of derivatives to easy problems in rates and in maxima and minima.

(c) Simple cases of inverse problems, e.g., finding distance from velocity, etc.

(d) Approximate methods of summation leading up to integration as a powerful method of summation.

(e) Applications to simple cases of motion, area, volume and pressure.

The work in calculus should be largely graphic and closely related to that in physics; the necessary technique should be reduced to a minimum by basing it wholly or mainly on algebraic polynomials. No formal study of analytic geometry need be presupposed beyond the plotting of simple graphs.

It is important to bear in mind that while the elementary calculus is sufficiently easy and interesting to justify its introduction, special pains should be taken to guard against any lack of thoroughness in the fundamentals of algebra and geometry. No possible gain could compensate for a real sacrifice of such thoroughness.

It should also be borne in mind that the suggestion of including elementary calculus is not intended for all schools nor for all teachers or all pupils in any school. It is not intended to connect in any direct way with college entrance requirements. The future college student will have ample opportunity for calculus later. The capable boy or girl who is not to have the college work ought not on that account to be prevented from learning something of the use of this powerful tool. The applications of elementary calculus to simple concrete problems are far more abundant and more interesting than those of algebra. The necessary technique is extremely simple. The subject is commonly taught in secondary schools in England, France and Germany and appropriate English texts are available.*

6. HISTORY AND BIOGRAPHY. Historical and biographical material should be used throughout to make the work more interesting and significant

At the present time these topics (1-5) can probably in most high schools be given most advantageously as separate units of a two-year program. However, the National Committee is of the opinion that methods of organization are being experimentally perfected whereby teachers will be enabled to present the same material more effectively in combined courses unified by one or more central ideas.

ADDITIONAL ELECTIVES

Such additional electives as *mathematics of investment, shop mathematics, surveying and navigation, descriptive or projective geometry* will appropriately be offered by schools which have special needs or conditions, but it seems unwise for the National Committee to attempt to define them pending the results of further experience on the part of these schools.

SUPPLEMENTARY NOTE ON THE CALCULUS AS A HIGH-SCHOOL SUBJECT

In connection with the recommendations concerning the calculus, such questions as the following may arise: Why should a college subject like this be added to a high school program? How can it be expected that high-school teachers will have the necessary training and attainments for teaching it? Will not the attempt to teach such a subject result in loss of thoroughness in earlier work? Will anything be gained beyond a mere smattering of the theory? Will the

*Quotations and typical problems from one of these texts will be found in a supplementary note appended to this report.

boy or girl ever use the information or training secured? The subsequent remarks are intended to answer such objections as these and to develop more fully the point of view of the Committee in recommending the inclusion of elementary work in the calculus in the high school program.

By the calculus we mean for the present purpose a study of rates of change. In nature all things change. How much do they change in a given time? How fast do they change? Do they increase or decrease? When does a changing quantity become largest or smallest? How can rates of changing quantities be compared?

These are some of the questions which lead us to study the elementary calculus. Without its essential principles these questions can not be answered with definiteness.

The following are a few of the specific replies that might be given in answer to the questions listed at the beginning of this note: The difficulties of the college calculus lie mainly outside the boundaries of the proposed work. The elements of the subject present less difficulty than many topics now offered in advanced algebra. It is not implied that in the near future many secondary-school teachers will have any occasion to teach the elementary calculus. It is the culminating subject in a series which only relatively strong schools will complete and only then for a selected group of students. In such schools there should always be teachers competent to teach the elementary calculus here intended. No superficial study of calculus should be regarded as justifying any substantial sacrifice of thoroughness. In the judgment of the Committee the introduction of elementary calculus necessarily includes sufficient algebra and geometry to compensate for whatever diversion of time from these subjects would be implied.

The calculus of the algebraic polynomial is so simple that a boy or girl who is capable of grasping the idea of limit, of slope, and of velocity, may in a brief time gain an outlook upon the field of mechanics and other exact sciences, and acquire a fair degree of facility in using one of the most powerful tools of mathematics, together with the capacity for solving a number of interesting problems. Moreover, the fundamental ideas involved, quite aside from their technical applications, will provide valuable training in understanding and analyzing quantitative relations—and such training is of value to everyone.

The following typical extracts from an English text intended for use in secondary schools may be quoted:

"It has been said that the calculus is that branch of mathematics which schoolboys understand and senior wranglers fail to comprehend . . . So long as the graphic treatment and practical applications of the calculus are kept in view, the subject is an extremely easy and attractive one. Boys can be taught the subject early in their mathematical career, and there is no part of their mathematical training that they enjoy better or which opens up to them wider fields of useful exploration. . . . The phenomena must first be known practically and then studied philosophically. To reverse the order of these processes is impossible."

The text in question, after an interesting historical sketch, deals with such problems as the following:

A train is going at the rate of 40 miles an hour. Represent this graphically.

At what rate is the length of the daylight increasing or decreasing on December 31, March 26, etc.? (From tabular data.)

A cart going at the rate of 5 miles per hour passes a milestone, and 14 minutes afterwards a bicycle, going in the same direction at 12 miles an hour, passes the same milestone. Find when and where the bicycle will overtake the cart.

A man has four miles of fencing wire and wishes to fence in a rectangular piece of prairie land through which a straight river flows, the bank of the stream being utilized as one side of the enclosure. How can he do this so as to enclose as much land as possible?

A circular tin canister closed at both ends has a surface area of 100 sq. cm. Find the greatest volume it can contain.

Postoffice regulations prescribe that the combined length and girth of a parcel must not exceed 6 feet. Find the maximum volume of a parcel whose shape is a prism with the ends square.

A pulley is fixed 15 feet above the ground, over which passes a rope 30 feet long with one end attached to a weight which can hang freely, and the other end is held by a man at a height of 3 feet from the ground. The man walks horizontally away from beneath the pulley at the rate of 3 feet per second. Find the rate at which the weight rises when it is 10 feet above the ground.

The pressure on the surface of a lake due to the atmosphere is known to be 14 lbs. per sq. in. The pressure in the liquid x inches below the surface is known to be given by the law $dp/dx = 0.036$. Find the pressure in the liquid at a depth of 10 feet.

The arch of a bridge is parabolic in form. It is 5 feet wide at the base and 5 feet high. Find the volume of water that passes through per second in a flood when the water is rushing at the rate of 10 feet per second.

A force of 20 tons compresses the spring buffer of a railway stop through 1 inch, and the force is always proportional to the compression produced. Find the work done by a train which compresses a pair of such stops through six inches.

These may illustrate the aims and point of view of the proposed work. It will be noted that not all of them involve calculus, but those that do not lead up to it.

Comments, criticisms and suggestions for the revision of this preliminary report are urgently requested. They should be sent as soon as possible to J. W. Young, chairman, Hanover, N. H.

SOME SUGGESTIONS ON ALGEBRA TEACHING IN TEXAS

GEORGE H. WELLS

The purpose of this article is to outline in detail the course in freshman and sophomore algebra as taught from the *New School Algebra* (Wentworth) in Texas and to suggest (1) certain methods of procedure in the classroom, (2) certain topics which in the author's opinion should be stressed and others which should be omitted or treated only briefly for the benefit of brighter pupils who are specializing in mathematics, and (3) some changes in the order and method of solution, all of which have been beneficial in classes taught by the author and in many others under his supervision. These changes and omissions will be given as they fit in with the course as outlined.

The first step in starting a new class in algebra is to acquaint them with the meaning, purpose, and practical value of the subject so as to arouse the interest and curiosity of the group. The first ten pages of the text should be used in this connection, supplemented to a large extent by the teacher. The next four pages dealing with the subject of parentheses may well be given later, following the discussion of subtraction and just before the work (page 54) on the insertion of parentheses.

The subject of simple equations (Chapter II) can be postponed with benefit and taught before taking up factors (Chapter VII). The average teacher finds the written problems more difficult for the class than the work on the fundamental operations (next chapters) and also that the relations of algebra and arithmetic as shown in the first chapter are more clearly followed up by the algebraic study of addition, etc. It has been the experience of many teachers that the work on written problems in simple equations, if taught first, tends to retard and discourage the weaker pupils, while they feel more "at home" if introduced to addition, subtraction, etc., immediately.

The algebraic operation in addition and subtraction of positive and negative numbers (Chapter III) should be carefully explained to the class by the line method of the text. The rules, after their derivation is carefully shown, should become by constant statement and application in class a matter of automatic use by the pupil. Reference to the reasons for the rules should not be made, except when the rule itself is not clear to the pupil. The problems in this chapter are all simple enough to be solved in class mentally and without previous assignment and preparation. The first half of this chapter may well be followed by the fuller study of addition and subtraction (Chapter IV) before multiplication and division are introduced. Here the problems are such as require pencil and paper and definite assignment. The immediate use of these problems will fix the rules of addition and subtraction firmly in the pupil's mind.

The methods of multiplication and division (Chapter III beginning page 42) should next be taught as in the text. The simple problems of this chapter should be solved mentally. A complete study of multiplication and division (Chapter V) may well follow. Special emphasis ought to be given long division as furnishing excellent drill in multiplication, division, and subtraction. This is a fine place in the course for a thorough review and a complete testing out of the class as to their ability to go on.

A study of the special rules of multiplication and division (Chapter VI) follows this review. The problems in this chapter should be as far as possible worked mentally in class. Care should be taken that every problem is worked by inspection methods.

The first semester's work for the average class includes a study of the type forms of factoring. These types may be called "the monomial factor type," "the grouping type," "the difference of squares type," "the trinomial type," "the cubes type," and "the factor theorem type." Special work should be given by exercises on miscellaneous examples to develop the student's ability to recognize these various types.

These exercises may be given with advantage without requiring solutions as merely a drill in telling how the problm should be attacked. In teaching the factoring of the trinomial, the most general form ax^2+bx+c (page 99) may well be taken up first, substituting for the "z" method of the text the "trial and error" method of many of the other good texts. Special study should be made of the signs of the trinomial and their effect upon the factors. The problems dealing with more special forms of trinomials (pp. 89, 97) can then be worked more quickly and more easily in this same way than by the special methods of the text. By following this order, the "trial and error" method can be made to include the solution of three or four forms with separate ways of solving. The advantage of decreasing the number of methods is apparent. The student at the same time gets from the special study of the trinomial a clearer conception of the formation of the quadratic equation. The application of this method to the "difference of squares type" ($a^2—b^2$, middle term $0x$) may be pointed out. In studying the "cubes type," a review of the rules for inspection division of cubes (p. 83) and their application to factoring will be useful.

The advance work of the class can end here (p. 103) and should be followed by a careful review of the semester's course with plenty of original problems worked in class and assigned.

The course for the second semester of the first year should begin with a good review of factoring (using the examples pp. 105-107). When the class learns to factor with facility they are ready to take up H.C.F. and L.C.M. (Chapter VIII). The finding of the H.C.F. and L.C.M. by factoring should be stressed and the Euclidian method (latter half of the chapter), if taught at all, should not be required of the weaker pupils. This method is hard to explain to pupils of freshman standing and training and is practically unnecessary if all the methods of factoring are carefully taught.

The subject of fractions (Chapter IX) should be taught next. Pupils should learn to make as many as possible of the simple operations—factoring, multiplying, collecting, etc. mentally. Each separate topic may well be preceded and illustrated by a review of the analogous operation in arithmetic. Care should be taken to avoid confusion of cancellation in multiplication with operations in addition and subtraction, so as to avoid the common error $\frac{x+3}{x} = 3$.

The study of fractional equations should immediately succeed the work on fractions. A review of the subject of simple equations (Chapter II) in class may be helpful at this time. Careful selection should be made of the written problems in this chapter, only a few of each kind being necessary. Further solution of the same type of problems becomes mechanical with the pupil. Special emphasis should be given the subject of formulae, introducing the formulae of physics, without scientific proof, instead of, or in addition to, the interest formulae of the text (p. 171). A thorough review of fractions at this point will probably give sufficient work for the remainder of the first year.

The work of the second year of the average class begins with the topic "simple simultaneous equations" (Chapter XI). The three methods of elimination should be taught with special stress on the substitution method as of value in quadratics. The problems requiring solution for three or more unknown quantities (p. 188) may well be given small allowance of time. The graph of the linear equation (Chapter XXVI to p. 381) should be introduced and this method of solution stressed. The written problems involving several unknowns (Chapter VII) follow. One or two problems only of each type should be assigned. Problems involving long discussions or having indeterminate solutions (especially pp. 203-4) should be omitted.

The subject of indeterminate equations and inequalities (Chapters XIII and XIV) may well be left out altogether or studied briefly at the close of the second year.

Involution and evolution (Chapter XV) are to be studied next. Emphasis ought to be given to square roots and the reasons for the method of finding the square root of numbers in arithmetic should be made clear by comparison with the process in algebra on compound expressions. Cube roots may well be given less attention as having but little value in future work either in high school mathematics or science. Some teachers advocate its entire omission.

The theory of exponents (Chapter XVI) should now be taught. The laws governing the operations on the exponent should be carefully taught, but their abstract proof (pp. 232-233) may be omitted without loss. Many of the problems in this chapter (especially p. 235) should be solved mentally.

The first term's work can close with a careful study of the operations on radical and imaginary expressions (Chapters XVII and XVIII). Special stress should be placed on the four fundamental operations on radicals and on the solution of equations containing radicals. Wherever possible in this chapter the problems should be worked orally. The subject of imaginary expressions should be taught very briefly and without the formal proofs (pp. 258-259).

The last semester's work in algebra should stress the solution of the quadratic (Chapter XIX). Solutions by means of the formula and by factoring may be emphasized relatively more than in the text and introduced earlier than in the text to advantage. Some teachers advocate teaching the "completing the square" method sufficiently only to enable the student to solve the general equation $ax^2+bx+c=0$ in deriving the formula. The use of the formula is very valuable not only as a method of solving the quadratic equation, but as a training in the use of formulae in general. If the subject of factoring, especially of the trinomial type, has been well taught to the class, the factoring method will be found easier, illuminating, and often permitting of mental solution. The "four times method" (p. 270) seems unnecessary and mechanical.

The solution of the quadratic equation leads at once to the study of simultaneous quadratic equations (Chapter XX). Here the simpler cases should be carefully taught but long involved problems having unusual solutions should be omitted. Classes often spend too much time on these problems (pp. 298-299). Emphasis may be given the substitution method of solution as carrying over from the solution of the linear equations. The solution of the quadratic by means of the graph (pp. 382-388) should be taught and the character of the curves briefly shown.

The study of arithmetical and geometrical progressions (Chapter XXII) is taken up now as given in the text. Emphasis should be given to the formulae and their application rather than to their derivation.

The subject of variables and limits (Chapter XXVIII) can well be given briefly in lecture form in sufficient detail to enable the pupil to understand the work in geometry requiring these notions.

The advance work in algebra ends with the study of the binomial theorem (Chapter XXV). The formula itself should be made clear and its application to simple problems worked out. This should be followed by a general review of algebra using simple type problems. The review of algebra should end with a thorough drill on the solution of the quadratic equation.

Having discussed the content and order of the course in algebra, the methods of handling this material in the classroom are of next importance. This part of the article is not intended as a complete discussion of classroom methods as applied to the teaching of algebra, but only as containing certain helps and suggestions which may be used by other teachers where they fit in with their individual methods.

In presenting a new subject in algebra, the rules governing it should as far as possible be worked out with the class by means of questions. A simple problem should be worked on the board by the instructor showing clearly the application of the rule. Pupils should be taught to ask questions freely here, and the teacher should patiently try to see that all understand the explanation. Careful watching of the faces will often tell if the procedure is clear. Following this, a group of problems may be worked by the pupil, mentally if possible, each pupil solving a problem showing all the steps in full. Two or more problems should be solved by the pupils to establish the rule firmly in mind before the assignment is given. This will help the pupil who understands in class but forgets the explanation of the teacher when the time for work and study outside of class comes. At the beginning of the next recitation period, time should be given as needed for questions about the work assigned and explained the day before. All problems should be made clear to the pupils by having them conquer their own difficulties under direction, rather than by having them worked by some other pupil who wishes to "show off." After a new topic or assignment has been introduced and several problems worked to illustrate it, a thorough drill on the day's assignment should be given and followed by miscellaneous problems from several days back.

Where mental drill is impossible, the work on the board is most satisfactory. An absolute routine for passing to and from the board, and getting and returning chalk and erasers on the way should be established. A definite place ought to be assigned each pupil. If the board space is too small and if the teacher has worked with the class for several weeks, the brighter pupils may be assigned to two or more others as monitors. They should not be allowed to correct mistakes but only to check them. This will keep the better pupils profitably occupied, help the weaker pupils, and enables the teacher to give more time to those who can not correct their errors.

Perhaps the greatest difficulty any teacher has is in training pupils to solve the written problems requiring the formation of, and then the solution of, an equation. Utmost care should be taken at the first in studying simple equations (Chapter II) that the method of translating English into algebra is made clear. A thorough exercise in the formation of equations from simple facts should precede the complete solution of these problems. Here the student should write out carefully every step. The student needs to learn the algebraic meaning of such words as "increased by," "diminished by," "exceeds." etc. They should also be taught to recognize the part of the equation which gives the equality between the two expressions, that is, if one quantity exceeds another by any number under certain conditions, that the two may be made equal by adding the difference to the smaller; that if a man gives money or goods to another he suffers a loss at the same time the other makes a gain. These facts, simple to the teacher, are often not thought out by the pupil. The pupil should see clearly what is given and what is to be found and the relation between the two. Rapid thinking often leads to carelessness and snap judgment by the class. If every step is at first put down and the reasoning worked out step by step, the student will develop habits of thought sufficiently clear to permit later of more mental work. The class should finally be able to write out the equation at once after stating what "x" stands for.

The difficulty of many classes in this as in other types of problems is that they can not read English understandingly. They need to be reminded that in mathematics every word counts and requires their careful attention. At first, advocate several readings of the problem before solution is attempted. Pupils are often too anxious to use their pencil and let it guide them. The use of Silent Reading Tests may point out to the teacher and pupil their cause of failure on these problems. Exercises increasing their ability along this line will then be of value.

In planning the course of study, in deciding on the parts of a text to be emphasized, and in working out a definite technique for classroom procedure, the teacher needs to do some careful thinking, continuous reading so as to keep up with the best ideas of the times, and to do some experimenting on a small scale to try out plans of his own. It is hoped that some of the suggestions of this article will prove of value in this connection, especially to the teachers of Texas, and more especially to those who are starting out in the work.

[Mr. Wells is Principal of the Cisco High School.]

ADVICE TO THE MAKERS OF QUESTION PAPERS

Final draft of a report prepared by a committee appointed at a conference called at the request of the Board of Review of the College Entrance Examination Board.*

1. *The Early Questions on Leading Topics.*

Fully one-third of the questions should be based on topics of such fundamental importance that they will have been thoroughly taught, carefully revised, and deeply impressed by effective drill. These questions must in the nature of the case be common-place in character. They should be of such a degree of difficulty that any pupil of regular attendance, faithful application, and even moderate ability may be expected to answer them satisfactorily.

2. *The Intermediate Questions.*

There should be both simple and difficult questions testing the candidate's ability to apply the principles of the subject. The early ones of the easy questions should be really easy for the candidate of good average ability who can do a little thinking under the stress of an examination; but even these questions should have genuine scientific content. Such questions as definitions in geometry and substituting numerical values in an algebraic formula, whether answered correctly or incorrectly, yield no substantial evidence concerning the candidate's quality or training.

It is not intended to exclude from the papers questions of a special or technical nature, as for instance the proof of the law of tangents in trigonometry or a book-work theorem in geometry which presents a peculiar difficulty. But such a question should not come early in the paper.

3. *The Hard Problem.*

There should be a substantial question which will put the candidates on their mettle, but which is not beyond the reach

*This committee was appointed by Dr. Farrand and consists of the following: W. F. Osgood (chairman); H. B. Fine, T. L. Bramhall, A. V. Galbraith, and H. B. Marsh.

of a fair proportion of the really good candidates. This question should test the normal workings of a well-trained mind. It should be capable of being thought out in the limited time of the examination. It should be a test of the candidate's grasp and insight—not a catch question or a question of unfamiliar character making extraordinary demands on the critical powers of the candidate, or one the solution of which depends on an inspiration. It may involve a novelty; it should not exploit a hobby or give way to an idiosyncracy of the examiners.

Above all, this question should lie near to the heart of the subject as the well prepared candidate understands the subject.

4. *Questions Direct and Homogeneous.*

The questions should be direct, and so put that the candidate readily sees precisely what is asked for. As a rule, a question should consist of a single part and be framed to test one thing—not pieced together out of several unrelated and perhaps unequally important parts. But a question may consist of several related parts, as when several expressions are given to be factored; and a single question on the paper may be made up of two questions, each a minor topic, as the the binomial theorem and the progressions.

Such composite questions should, however, be used sparingly. The important topics should be represented by questions so substantial as to deserve and to receive each an individual number. Thus the candidate can see readily what progress he is making with the paper, and the readers can mark the paper according to an intelligible plan.

5. *Relative Weight of the Questions.*

Each question should be a substantial test on the topic or topics which it represents. It is, however, in the nature of the case impossible that all questions be of equal value. Most candidates will do the easy questions first, and as much on the hard questions as they have time and ability for. But what better use could a candidate make of the paper than, after getting warmed up on a few of the easier questions, then to turn to a hard one and give a clean-cut solution of it? If he thus omits an easy question, is his book to receive the same mark as that of a candidate who does the easy questions and omits the hard ones? The readers are explicitly instructed to deal with such cases in a fair and appreciative manner, and they are free to use the method of the bonus in estimating the value of such a book and determining the mark to be assigned to it. The makers of the question paper must not feel obliged to attempt to even up the numbered questions so that the readers can mark them mechanically, for the latter course is explicitly disavowed by the Board.

6. *Length of the Paper.*

Care should be used that the examination be not too long. Reasonable facility in answering questions under the conditions of an examination may fairly be expected of the candidate. But he should not be confronted with a paper which he feels he can not finish if he stops to think before writing down his answer, in order to prepare the form as well as the substance of what he has to say. The examiner must not feel it his duty to cover all, or even most of the minor topics on every paper. He should be content to ask questions on the important topics, so chosen that their answers will be fair to the candidate and instructive to the readers; and beyond this merely to sample the candidate's knowledge on the minor topics. Thus the readers will be able to give credit for answers systematically thought out and carefully executed. *Non multa, sed multum,* should be the guiding principle.

It is better to have too short rather than too long a paper. Experience shows that a paper, in order to be fair to the candidate should look too easy to the examiners.

7. *The Paper as a Whole—Pit-Falls for the Examiners.*

The paper should be considered as a whole with reference to the way in which it is likely to strike the candidates. It is not enough to consider the individual questions with reference to their suitability. No reasonable question, detached, is either suitable or unsuitable. All depends on its relation to its fellows. A paper may be made up of questions each of which, in a conceivable setting, would be admirable, and yet the paper, taken as a whole may present a series of difficulties the cumulative effect of which is to make the examination ineffective for the main purpose for which examinations exist, namely, to enable the candidate to demonstrate his knowledge of the subject and his ability to apply its principles.

Moreover, the examiners must bear in mind that the character of the papers set by such a body as the Board can not be in all respects the same as the tests one uses in one's own classes. There, the pupil or student has ample opportunity to become familiar with the personal equation of the examiner. It is as if a nine were always to face the same pitcher. The examiners should, therefore, take especial care that the questions correspond to that which is universal in the subject.

There are subtle defects, the effect of which may be serious. First, the language of a question may present difficulty to the child's hand. Second, a paper may lay undue stress on a special topic or method, as the paper in geometry which ran to tangents and the paper in algebra which ran to factoring. Third, a question may be such that an error in understanding what is asked for or in the method of solution may be exceedingly costly in time for the candidate. It is impossible to be exhaustive on this subject. It is full of pit-falls for the examiners, the penalty for which is paid by the candidates. It is the most difficult part of the making of the question papers, and the part which hitherto has received the least attention.

8. *Procedure of the Committee.*

A plan which some committees have used with success is as follows. The chairman assigns to each member of the committee a paper on a leading subject. The members prepare their papers and each sends a copy of his paper to every member of the committee. The members criticize each other's papers before the meeting, if they choose, and each member comes to the meeting with definite knowledge of the papers which are to be discussed.

Whatever plan is adopted, the examiners should take ample time to examine the individual papers before the first meeting of the committee is held. Each paper which involves computation should have been worked in detail by at least two members of the committee, who will report to the committee the time it took them. Furthermore, the committee should scrutinize with care the language of the questions, with respect both to clearness and to good English.

Only by exercise of the most painstaking care on the part of each member of the committee can the examination be made a suitable test for the candidates. Shortcomings on the paper which appear trivial to the layman are paid for heavily by the candidates in a disproportionate mortality, and the result is loss of faith in the fairness and value of the examinations of the Board.

A FEW SUGGESTIONS TO TEACHERS OF ELEMENTARY ALGEBRA

IRVING BALL

The question of what should be included in the algebra taught in high school and what should be omitted has been discussed at length many times with but little progress toward a definite settlement. It seems to me that the real question that presents itself to the teacher of algebra is not so much, "What shall I teach?" as, "How shall I teach?"

The purpose of this paper is not to build up an impractical theory, but to give suggestions that will be of real value to algebra teachers, especially young teachers who are seeking better methods. The following suggestions, which are offered, come to the writer as the results of his own experience in teaching algebra and from his observation of the teaching of others.

1. Review the principles of arithmetic as they come up in algebra. Teachers of algebra, in common with teachers of other subjects, assume that their students know more than they actually do. We take it for granted that because a boy has "completed" the arithmetic, he has a thorough foundation in and a working knowledge of all the principles contained therein, but experience shows that most beginners in algebra have not mastered and generalized the principles of arithmetic to such a degree that they can apply these principles to abstract problems. To make use of a simple illustration: When beginners are asked what a apples cost at c cents each, they do not readily see that the answer is ac cents, because they have not thoroughly mastered the principle that the cost of a number of articles equals the number bought times the cost of one; they are slow to understand that the interest of p dollars at r per cent for t years is $prt/100$, because they have not generalized the principle that the interest of a sum of money is equal to the number of years times the rate expresed in hundredths times the principal; they have, in all probability, forgotten that to multiply

a fraction by a whole number, they must multiply the numerator only and not both terms. The same lack of ability to generalize many principles, the same failure to remember others will be found all along the line and the wise teacher will spend sufficient time in reviewing these principles as they are needed.

2. Many teachers allow too indefinite and too slipshod statements in the solution of the so-called written problems. The pupil seems to have special difficulty with these problems and many times fails to solve them simply because he does not have a clear idea of exactly what x stands for. For example, in solving the problem, "A man pays $300 for a horse and a cow, paying twice as much for the horse as for the cow; how much does he pay for each?"—many pupils start off bravely with, "Let x=the cow, and $2x$=the horse" and then are all at sea as to how to proceed. Teachers who tolerate such statements need not expect to reap an abundant harvest in developing pupils of power. Boys and girls should be trained to think in terms of precisely what x does equal. Mathematics is an exact science and should be so taught.

3. Much difficulty in problem-solving is due to the fact that students do not thoroughly comprehend the meaning of a problem before attempting a solution. Occasionally, they even ask the teacher for help and, on being questioned, confess that they actually have not yet read the problem through, but had started in with, "Let x=something or other" with but a very hazy idea of what it was all about. Pupils should be trained to make analysis of a problem before attempting a solution. In my opinion, much time—even a whole lesson now and then—may be profitably spent in problem analysis. When attempting a lesson of this character, individual pupils should read the problem, then after sufficient time is given for him to determine what it means, let him state as concisely as posible, but without hurrying, just what steps he would take in its solution. When the first problem has been thus analyzed, let another pupil proceed in like manner with the second, and so on. I believe it is a fine thing to let each pupil state also about how much he

thinks the answer should be. This develops his judgment and in time will lead him to reject incongruous results of his own solutions and go back over them for errors. If pupils are trained to analyze problems in this way, there will be less of the discouraging hit-or-miss habit of first multiplying and then, if that doesn't bring the answer, dividing.

4. The mere mechanical solution of a problem is by no means evidence that a pupil understands it. I have had a pupil, after he had proudly explained the mechanical solution of some problem, look at me in blank astonishment at my innocent request that he show that his result met the conditions of the problem. After a little, he generally begins to substitute for x in the equation used in the solution, not realizing that this operation is a check for the result only when his equation is the correct one. The pupil should thoroughly understand this point and should be taught to show that his results meet all the conditions of the problem without any reference whatever to the letter he let stand for the unknown. A pupil can never be sure he can solve a problem until he can do this. Thoroughness here takes more time at first, but is made up many times in increased efficiency later. We want to develop pupils who know and who know that they know.

5. Principles should generally be developed inductively and then special drill given to apply them. Pupils understand a principle much better if they develop it themselves, stating it first in their own words and later in the better language of the book. Some teachers erroneously claim that they do not have time for this method, when the truth is they can not afford to dispense with it, as this sort of teaching leads a pupil to become more and more independent and less dependent upon his teachers. It has been my observation that more students fail in algebra because they have not been thoroughly taught the principles than for any other one reason. Early last fall I had been urging my teachers to do more inductive teaching in algebra and other subjects and, by way of experiment, brought a few volunteer pupils from the eighth

grade algebra class to our faculty meeting one day. The class had had none of the special rules of multiplication and I determined to develop the rule for the product of problems of the type $(x+a)(x+b)$. After these boys and girls had found the products of many problems of this type by actual multiplication, I started them on the fun of trying to find a rule by which they could write out any of these products without actual multiplication. It took some time, but after a while success crowned their efforts and they were the happiest youngsters in the world to know that they had made a discovery. It is the pupil's right to be given every opportunity possible to think and to be given this pleasure in discovery.

[Mr. Ball is Superintendent of the Quanah Public Schools.]

MOTIVATION IN THE TEACHING OF MATHEMATICS

MARGARET BREWER

While mathematics instruction is rated as admirable disciplinary procedure, yet we should be impressed with the extremely practical side of mathematics work and we should see to it that an appeal is made to the student through the proper regard for motives in the selection of subject matter.

As is well known, work in mathematics is well adapted to competition. This, perhaps, has been one of the most easily recognized motives—perhaps the strongest motive—in the mind of the student of mathematics in the public schools. This is often *the motive* which furnishes the necessary impulse to go through with the drill and drudgery of mathematics work. The work in mathematics being simple and easily organized, it is easy for the teacher to keep students constantly succeeding and advancing—constantly scoring additional gains. This should prevent discouragement and makes failures in mathematics unusual. But there are many even greater possibilities for the wide-awake teacher of the subject.

Situations should be introduced which will motivate even the drill work. Set a problem which will fully cover the ground of the assignment—which will include every element of the formal object of the lesson—and which will also include for the pupil a larger interest, a more personal quest. This larger interest on the part of the child will provide the necessary motive for the work. It will put more of the "something to do" factor in the pupil's assignment.

Still another source of motivation in mathematics instruction is found in its practical application to other school work. Vocational subjects, science subjects, and even history are rich fields for the application of knowledge gained in mathematics study. Athletics also provides additional incentives for the application of mathematical processes. The school bank is another activity that can be used to advantage by the mathematics department. To this list of school work that provides motivation for mathematics work could be added a long list of individual projects and problems of the individual student upon which the wise teacher will draw without hesitation.

When to this list is added the possibilities of the imaginary problem it can readily be seen that thorough motivation can be had without in any sense destroying the unity of the subject, and without sacrificing it to other subjects which are really no more important than mathematics. Nor is it at all necessary to eliminate or even to reduce the amount of drill—which is necessary if proper results are to follow your instruction. Neither will it prevent the development of speed and accuracy which are necessary if the work is to be of any practical value when actual problems are to be solved by the pupil in the school of life.

The introduction of occupational projects into the mathematics work is a field of almost limitless possibilities as to subject-matter and interest. It is an opportunity to introduce the child to the world of industry and achievement, of science and invention, of commercial and financial structures, of forestry and mining, of civics and art. The world at work is the field fom which the teacher of mathematics may draw material for instruction, for inspiration, for motivation in her work. And while she is doing this she is probably lending a hand in the game of making of the student a keener observer of the world of affairs, of the nation at work, of his community and its industries. She is helping to break down the fancy that mathematics is impractical and useless. She is doing the one big thing that mathematics can do better perhaps than any other subject: helping the individual to learn how to think a thing through to a correct conclusion, and at the same time she is quickening his latent spirit of citizenship and achievement in the community and in the world of affairs.

[Miss Brewer is the mathematics teacher in the Terrell High School.]

BROWN FRESHMAN PRIZE EXAMINATION

H. J. ETTLINGER

At the beginning of every academic year, the Department of Pure Mathematics holds a prize examination for freshmen. Through the generosity of an alumnus of Brown University, it is possible to offer three prizes of fifteen, ten and five dollars, respectively, for the three best examinations on the elementary algebra of the first two years of high school and a year of plane geometry. The competition is open to students just entering the freshman class of the University of Texas, and the ground covered is that of the minimum entrance requirements in mathematics.

The examination was held this year on October 15, and lasted one hour. There were thirty competitors and fifteen handed in papers. It very often happens that the best showing is made by students having only the minimum requirements and no more. In the past the prize winners have been from the smaller schools of the state. This year Austin carried off the first two prizes and Laredo the third. The prize winners in order were Howard Ferguson, Robert R. Brown, and Rosalie Biggio.

The questions were the following:

1. In buying coal A gets 3 tons more for \$135 than B does and pays \$7 less for 4 tons than B pays for 5. Required the price each pays per ton.

2. At his usual rate a man can row 15 miles downstream in 5 hours less time than it takes him to return. Could he double his rate, his time downstream would be only 1 hour less than his time upstream. What is his usual rate in still water and what is the rate of the current?

3. Given the lengths of the three altitudes of a triangle, to construct the triangle.

4. To find upon a given straight line AB a point M such that the straight lines which join it to the vertices, D and E,

of a given triangle DEF form with the side DE a triangle DEM whose area is one-half as great as that of the given triangle.

It will be of interest to the teachers in our high schools to try these problems on their good students. The department will be glad to furnish solutions.